

Dissipative effect of thermal radiation loss in high-temperature dense plasmas

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A dynamical model based on the two-fluid dynamical equations with energy generation and loss is obtained and used to investigate the self-generated magnetic fields in high-temperature dense plasmas such as the solar core. The self-generation of magnetic fields might be looked at as a self-organization-type behavior of stochastic thermal radiation fields, as expected for an open dissipative system according to Prigogine's theory of dissipative structures.

Thermal motion of electrons generates thermal radiation mainly by means of thermal bremsstrahlung emission in high-temperature plasmas. The higher the plasma temperature the more intense thermal radiation of the plasma. Thermal radiation causes substantial energy loss in high-temperature plasmas such as thermal nuclear fusion plasmas; for instance, the well-known Lawson condition is derived from the condition that the fusion energy compensates the thermal bremsstrahlung emission loss. Here we are about to report some dissipative effect of thermal radiation loss in high-temperature dense plasmas such as thermal nuclear fusion plasmas.

Generally, the elastic collision frequency ν_{ei} between electrons and ion is high in high-temperature dense plasmas, for example, in the core of the Sun $T_e \sim 10^7$ K, $n_e \sim 10^{32} \text{ m}^{-3}$, we can estimate $\nu_{ei} \sim 10^{16}$ Hz, the plasma frequency $\nu_{pe} \sim 10^{17}$ Hz. Such a frequent collision would destroy any collective plasma motion or coherent structure if the Sun were isolated adiabatically. In other word, the Sun would be in thermodynamic equilibrium if it was an isolated system. However, the Sun is in fact an open system: it not only gains thermal energy from the inelastic collisions between ions such as the proton-proton chain (or pp chain), but also loses energy by means of thermal radiation, for example, the loss rate of thermal radiation or the photon luminosity $L_\odot \sim 4 \cdot 10^{33} \text{ erg/s}$ (Bahcall, 1989). Therefore, the Sun may be far from equilibrium. The theory of dissipative structures (Prigogine, 1973; 1977) has shown that coherent behaviors may occur in a dissipative system. In fact, we have witnessed and are witnessing the dissipative structures in the convective zone of the Sun, the convective cells. These are the coherent structures at the outer part of the Sun caused by thermal radiation dissipation which maintains the necessary temperature gradient. What are the dissipative structures in the source region?

Roughly, nuclear fusion reactions take place within $0.3R_\odot$ (Bahcall, 1989), where nuclear reactions (inelastic collisions between ions) first enhance stochastic motion of ions, the ions then transfer energy and momentum to electrons via elastic collision. The electrons lose energy and momentum through emitting photons via inelastic

collision with ions and the thermal radiation escapes from the source region via radiation transfer. On average, the direction of the energy and momentum flux in the nuclear fusion region is therefore:

$$\text{ions} \rightarrow \text{electrons} \rightarrow \text{photons} \quad (1)$$

This implies that electrons do not reach thermodynamic equilibrium with ions ($T_i > T_e$), nor do photons with electrons in the fusion plasma and that it is the very ions that replenish the radiation loss of both energy and momentum of electrons.

This nonequilibrium effect also shows itself in the two-fluid dynamical equations (Rose and Clark, 1961; Ma *et al*, 1988; Rybicki and Lightman, 1979):

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = \frac{\delta n_\alpha}{\delta t} \quad (2)$$

$$m_\alpha n_\alpha \frac{d\mathbf{V}_\alpha}{dt} = -\nabla \cdot \mathbf{P}_\alpha + n_\alpha \mathbf{F}_\alpha + \mathbf{R}_\alpha + \delta \mathbf{R}_\alpha \quad (3)$$

$$\frac{3}{2} n_\alpha \frac{dT_\alpha}{dt} = -(\mathbf{P}_\alpha \cdot \nabla) \cdot \mathbf{V}_\alpha - \nabla \cdot \mathbf{q}_\alpha + Q_\alpha + \delta Q_\alpha \quad (4)$$

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad (5)$$

where ds is a differential element of length along the ray, I_ν is the energy crossing unit area in unit time, unit solid angle and unit frequency range of radiation fields, called as the specific intensity or brightness, α_ν and j_ν are the absorption coefficient and emission coefficient respectively. \mathbf{P}_α is called the kinetic stress tensor, \mathbf{q}_α the thermal flux vector, \mathbf{F}_α is the total force exerted on the fluid element of α species, Of five collision-related terms, $\delta n_\alpha / \delta t$ represents created or destroyed particle density of α species in the unit time, \mathbf{R}_α is the momentum gain or loss rate of the fluid element of α particles by their elastic collision with the unlike particles (including photons), the same for $\delta \mathbf{R}_\alpha$ but due to the inelastic collision. Similarly, Q_α and δQ_α are thermal energy gain or loss rate due to the elastic collision and the inelastic collision respectively.

For the sake of clarity, we assume that the quasi-stationary state has been reached with $\partial T_\alpha / \partial t \approx 0$, as believed in the solar core. If we further neglect viscosity, thermal conduction, thermal convection and restrict to a very short spacescale ($< 1cm$), the energy conservation equation reduces to

$$0 = Q_\alpha + \delta Q_\alpha \quad (6)$$

Eq.(5) tells us that radiation energy is continuously transferred from the nuclear fusion region to the outside due to the radiation loss, which implies that electrons lose energy through emitting photon via inelastic collision with ions, $\delta Q_e < 0$, Eq.(6) thus leads to $Q_e = -\delta Q_e > 0$. This means that electrons should gain energy on average from ions via elastic collision with ions. In the quasi-stationary case, $Q_i \equiv \sum_{\alpha \neq e} Q_\alpha = -Q_e < 0$, which implies that ions lose energy on average via elastic collisions with electrons. The energy loss of ions $Q_i < 0$ will be replenished by the energy generation $\delta Q_i > 0$ due to the inelastic collision (i.e., nuclear reaction) between ions or ions and electrons, $Q_i = -\delta Q_i$, as implied by the energy conservation equations for ions. The energy transfer accompanies similar momentum transfer between electrons and ions: $\mathbf{R}_e = -\mathbf{R}_i \equiv \sum_{\alpha \neq e} \mathbf{R}_\alpha > 0$, which means that ions lose momentum and electrons gain momentum on average in the elastic collision between ions and electrons in the quasi-stationary case. The momentum generation $\delta \mathbf{R}_i \equiv \sum_{\alpha \neq e} \delta \mathbf{R}_\alpha > 0$ of ions due to the nuclear fusion collision replenishes the momentum loss $\mathbf{R}_i < 0$ of ions.

The following task is to estimate $\mathbf{R}_i + \delta \mathbf{R}_i$ and $\mathbf{R}_e + \delta \mathbf{R}_e$. In so doing we have to take into account the radiation fields. The transfer equation (Rybicki and Lightman, 1979) takes a particularly simple form if, instead of s , we use the optical depth τ_ν defined by $d\tau_\nu = \alpha_\nu ds$,

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + j_\nu / \alpha_\nu \quad (7)$$

In plane-parallel media, a standard optical depth is sometimes used to measure distance normal to the surface, so that $\tau_\nu = \tau_\nu(z)$. A medium is said to be optically thick or opaque when τ_ν , integrated along a typical path through the medium, satisfies $\tau_\nu > 1$. An optically thick medium is one in which the average photon of frequency ν cannot traverse the entire medium without being absorbed. Solar core plasma is optically thick, and the absorption is mainly due to the (massive) ions (Bacall, 1989). If a medium absorbs radiation, then the radiation exerts a force on the medium, because radiation carries momentum. The radiation pressure \mathbf{F}_{rad} can be defined by the specific intensity I_ν in remembering that a photon has momentum $h\nu/c$,

$$\mathbf{F}_{rad} = \frac{1}{c} \int \alpha_\nu I_\nu \mathbf{n} d\Omega d\nu \quad (8)$$

where \mathbf{n} is a unit vector along the direction of the ray,

z -direction, for instance. Since the absorption is mainly due to the ions, we know

$$\mathbf{R}_i + \delta \mathbf{R}_i = \mathbf{F}_{rad} \quad (9)$$

$$\mathbf{R}_e + \delta \mathbf{R}_e \approx 0 \quad (10)$$

As a result, the momentum transport equation reduces to

$$m_i n_i \frac{d\mathbf{V}_i}{dt} = -\nabla p_i + n_i \mathbf{F}_i + \mathbf{F}_{rad} \quad (11)$$

$$m_e n_e \frac{d\mathbf{V}_e}{dt} = -\nabla p_e + n_e \mathbf{F}_e \quad (12)$$

where m_i , n_i and \mathbf{V}_i are the effective mass, density of ions and the effective velocity of an ion-fluid element. This is in agreement with the nonequilibrium processes (1). The most outstanding characteristic of the reduced equation is that the collision terms have been canceled out. In contrast, the cancellation cannot occur in the equilibrium case because there is no a well-determined direction of energy and momentum flux. Consequently, we find out that the nonequilibrium effect cancels the collision effect.

In the two-fluid approximation $\mathbf{V}_\alpha = \mathbf{V}_\alpha(\mathbf{r}, t)$ is the velocity of an α fluid element, which is the first-order moment of the distribution function at the velocity space of the α particle. Under such an approximation, the acceleration behavior of single particles has been ruled out. Therefore, the Maxwell equations (Rose and Clark, 1961; Ma *et al*, 1988)

$$M(\mathbf{E}, \mathbf{B}; \rho, \mathbf{J}) = 0 \quad (13)$$

with induced charge density $\rho = \sum_\alpha q_\alpha n_\alpha$ and induced current density $\mathbf{J} = \sum_\alpha n_\alpha q_\alpha \mathbf{V}_\alpha$ determine only the induced fields in the plasma by motion of fluid elements. The radiation fields \mathbf{E}_{rad} and \mathbf{B}_{rad} are generated by the acceleration motion of single particles and thus can be determined only by the retarded potentials of single moving charges, or Liénard-Wiechart potentials. On averaging single particle fields \mathbf{E}_{sin} and \mathbf{B}_{sin} in the velocity space with the distribution function of electrons as the weight factor, we can obtain the radiation fields (Rybicki and Lightman, 1979). At this stage, we are able to write down explicitly

$$\mathbf{F}_\alpha = q_\alpha [(\mathbf{E} + \mathbf{E}_{rad}) + \mathbf{V}_\alpha \times (\mathbf{B} + \mathbf{B}_{rad})] + \mathbf{g}_\alpha \quad (14)$$

where \mathbf{g}_α is gravity, $\alpha = i, e$. The quasi-stationary condition requires that the thermal and radiation pressure balance the gravity on the macroscopic level ($\gg 1cm$),

$$-\nabla p_i + \mathbf{F}_{rad} + \mathbf{g}_i = 0 \quad (15)$$

$$-\nabla p_e + \mathbf{g}_e = 0 \quad (16)$$

As a result, Eqs.(11) and (12) reduce to

$$m_\alpha n_\alpha \frac{d\mathbf{V}_\alpha}{dt} = -\gamma_\alpha T_\alpha \nabla \tilde{n}_\alpha + n_\alpha q_\alpha (\mathbf{E}^{tot} + \mathbf{V}_\alpha \times \mathbf{B}^{tot}) \quad (17)$$

where \tilde{n}_α ($\alpha = i, e$) is the microscopic density fluctuation of the α fluid. $\mathbf{E}^{tot} = \mathbf{E} + \mathbf{E}_{rad}$, the same for \mathbf{B}^{tot} . We have assumed thermal pressure $p_\alpha = \gamma_\alpha n_\alpha T_\alpha$ with the polytropic index γ_α . The mass conservation equation can be rewritten down as follows by neglecting particle creation or destruction:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0 \quad (18)$$

So far we have shown that the plasma in the nuclear fusion region can approximately be described by Eqs.(13), (17) and (18) when the energy and momentum flux (1) is well-defined by the thermal radiation loss. These equations possess two characteristics: (a) The elastic collision terms have been canceled out by the inelastic collision terms; (b) thermal radiation fields only appear in the momentum transport equations. The first confirms the fact that non-equilibrium may be a source of order, the second allows us to study the self-organization phenomena (Nicolis and Prigogine, 1977) of the random thermal radiation fields. Self-organization will lead to order, or dissipative structures.

Self-organization needs nonlinearity. The reduced two-fluid dynamical equations contain the needed nonlinearity. These equations can be simplified substantially through distinguishing the ion-timescale $\tau_i \sim \omega_{pi}^{-1}$ and the electron-timescale $\tau_e \sim \omega_{pe}^{-1}$ with the self-generated effect of magnetic fields included in (Kono *et al.*, 1981; Li, 1993a,b):

$$i \frac{\partial \mathbf{E}}{\partial t} + \alpha \frac{\partial^2 \mathbf{E}}{\partial z^2} - (\beta z + pn) \mathbf{E} + ip \mathbf{E} \times \mathbf{B} = \mathbf{S} \quad (19)$$

$$i \frac{\partial E_z}{\partial t} + \frac{\partial^2 E_z}{\partial z^2} - (\beta z + pn) E_z + ip [\mathbf{E} \times \mathbf{B}]_z = S_z \quad (20)$$

$$(\frac{\partial^2}{\partial t^2} + 2\nu_i * \frac{\partial}{\partial t} - \gamma \frac{\partial^2}{\partial z^2}) n = \gamma' \frac{\partial^2}{\partial z^2} |\mathbf{E}|^2 \quad (21)$$

where \mathbf{E} is the slowly varying complex amplitude of the induced high-frequency (ω_0) electric field $\tilde{\mathbf{E}}$: $\tilde{\mathbf{E}} = \frac{1}{2} \{ \mathbf{E} \exp(-i\omega_0 t) + c.c. \}$. The plasma has been assumed to be plane-parallel to the density gradient of scalelength L along z -axis. Therefore, all variables depend on only z . Then, $\mathbf{k} = (0, 0, k)$. $\mathbf{E}^t = [E_x, E_y]$ is the transverse field component, while E_z is the longitudinal field component. The validity conditions of these equations are $k/k_{De} \ll 1$, $Wk/k_{De} \ll 1$ and $|\delta n|/n_e \leq \delta\nu/\nu_{pe} \ll 1$. Another physical constraint is $\delta n/n_e \leq \delta\nu/\nu_{pe}$. All variables have been rescaled as follows (Morale and Lee,

1977): $\mathbf{B} = \tilde{\mathbf{B}}_s/B_{ML}$, $\mathbf{E} = \tilde{\mathbf{E}}/E_{ML}$, $n = (\delta n/n_e)/N_{ML}$, $t = \tilde{t}/T_{ML}$ and $z = \tilde{z}/Z_{ML}$, in which the tilded stand for the unrescaled. $B_{ML} = \beta_1 a^2 W$, $E_{ML} = 2\beta_1 a \tilde{E}_0$, $N_{ML} = \beta_1 a^2 W$, $T_{ML} = 2a/\beta_1 \omega_{pe}$ and $Z_{ML} = a^{-1} L/\sqrt{\beta_1}$ are the new units, in which $a = (L\omega_{pe}/\sqrt{3}V_{Te})^{2/3}$, $\beta_1 = 1 + 3(T_i/T_e)$, $W = \tilde{E}_0^2/(n_e \kappa T_e/\epsilon_0)$. The other parameters are: $\alpha = (c/V_{Te})^2/3$, $\beta = \beta_1^3$, $p = a^3 W/(1 - B_0^2)$ in that $B_0 = \tilde{B}_s/(m_e \omega_{pe}/e)$, $\gamma = \frac{4}{3} a^3 (m_e/m_i)$, $\gamma' = \gamma/(1 - B_0^2)^2$, $S_x = (1/2\beta_1^2) \cos \phi \sin \theta$, $S_y = (1/2\beta_1^2) \sin \phi \sin \theta$ and $S_z = (1/2\beta_1^2) \cos \theta$ in that both $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$ are random due to random orientation of the thermal radiation field. $\tilde{E}_0 \approx [\rho(\nu_{pe}, T) \delta\nu/\epsilon_0]^{1/2}$ is the thermal radiation field components at and near the local plasma frequency ν_{pe} , where $\rho(\nu, T)$ is spectral energy density of thermal radiation. The symbol “*” means convolution product, ν_i is the Landau damping of low-frequency longitudinal waves.

The so-called self-generated magnetic field $\tilde{\mathbf{B}}_s$ of the plasma is the slowly varying component of the induced magnetic field (Kono *et al.*, 1981):

$$\frac{\tilde{\mathbf{B}}_s}{B_c} = (\frac{V_{Te}}{c})^2 \frac{i\tilde{\mathbf{E}} \times \tilde{\mathbf{E}}^*}{E_c^2} \quad (22)$$

where $B_c = m_e \omega_{pe}/e$ is the critical magnetic field at which electron gyrofrequency $\Omega_e = eB_c/m_e$ equals to local plasma (angular) frequency ω_{pe} , $E_c = (n_e \kappa T/\epsilon_0)^{1/2}$. The magnetic-field generation was shown to be due to a solenoidal current $\mathbf{j}_s = -ie\omega_{pe}^2/(16\pi m_e \omega_0^3) \nabla \times (\mathbf{E} \times \mathbf{E}^*)$ [Eq.(1.1) of Kono *et al.*, 1981] by many authors (see Kono *et al.*, 1981 and references cited therein). Through taking the curl of the solw-timescale Ampère law and neglecting the displacement current, one obtains [Eq.(4.7) of Kono *et al.*, 1981]

$$\nabla \times \nabla \times \tilde{\mathbf{B}}_s = \frac{4\pi}{c} \nabla \times \mathbf{j}_s \quad (23)$$

This equation reduces to Eq.(22) when $\nabla \cdot \tilde{\mathbf{B}}_s = 0$ is satisfied, as one of the Maxwell equations requires. Eq.(22) shows that if there is no self-organization of the stochastic thermal radiation field \mathbf{E}_{rad} , which has been incorporated in parameters \mathbf{S} and p , the self-generated magnetic field $\tilde{\mathbf{B}}_s$ must be very weak, vice versa. Therefore, we can investigate the self-organization effect of the thermal radiation field through monitoring $\tilde{\mathbf{B}}_s$. In order to make sure that we have observed the self-organization phenomenon, we have to use the statistical average $\tilde{B}_0(\tilde{z})$ of $\tilde{B}_0(z', \tilde{t})$ over time, space and random initial conditions:

$$\tilde{B}_0(\tilde{z}) = \frac{1}{n-2} \left(\sum_{i=1}^n \bar{B}_i - \bar{B}_{max} - \bar{B}_{min} \right) \quad (24)$$

where $\tilde{B}_i = L_{sim}^{-1} \{ \int_z^{z+L_{sim}} [\tilde{B}_x^2(z') + \tilde{B}_y^2(z')] dz' \}^{1/2}$ and n is the number of solutions starting from random initial conditions. \bar{B} means averaging $\tilde{\mathbf{B}}$ in the range

$\tilde{B}_{max}/B_c \in [0.8, 0.9]$, where $\tilde{B}_{max} = \text{Max}\{[\tilde{B}_x^2(z) + \tilde{B}_y^2(z)]^{1/2}\} \leq 0.9B_c$ is one of our controlling parameters (the other is the maximum density fluctuation $\delta n/n_e \leq \delta\nu/\nu$). L_{sim} is the simulation cell size.

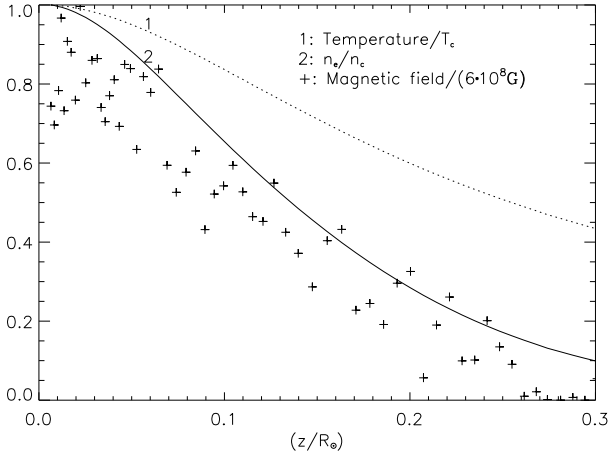


FIG. 1. Variation of the self-generated magnetic field with the solar radius coordinate z/R_\odot , where R_\odot is solar radius. Curves 1 and 2, representing solar plasma temperature and electron number density respectively, are the input parameters. Symbols “+” stand for the corresponding self-generated magnetic fields.

A standard second-order explicit quasispectral method (Li and Li, 1993; Li, 1996) has been used to numerically solve the model equations in order to calculate B_x and B_y (note: $B_z = 0$) and thus \tilde{B}_s . We have used table XVI of Bahcall & Pinsonneault’s standard solar model (1992) to calculate our input parameters: the electron and ion temperatures $T_e \approx T_i = T$, electron number density $n_e = \frac{1}{2}(1+X)\rho/m_H$, and plasma scalelength $L = n_e(dn_e/dz)^{-1}$, where $X = X(^1H) + X(^3He)$ represents the concentration, by mass, of hydrogen. Although the simulation cells are small, $L_{sim} \sim 10^{-3}$ m, it is much larger than the Debye length $\lambda_{De} = V_{Te}/\omega_{pe} \sim 10^{-11}$ m. Figure 1 shows $\tilde{B}_0(z)$ obtained by using $n = 10$ (the convergence trend has been found by using $n = 1, 4, 8$ and 10), which shows at least 0.4 gigagauss magnetic field may be generated at the center of the Sun. Such a strong field shows that the self-organization behavior of the stochastic thermal radiation fields does occur. The collision time scale is about 10^{-16} s, while the growth time scale of the self-generated magnetic field is about 10^{-12} s. The fact that the coherent time scale is much longer than the collision time scale is also an indicator of self-organization (Nicolis and Prigogine, 1977).

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